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SMALL DECISION MAKING UNDER UNCERTAINTY AND RISK

Dr. Takemi Fujikawa

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ABSTRACT

Small Decision Making under Uncertainty and Risk [1]

We everyday engage in the "Small Decision Making (SDM)". This paper presents an experiment on SDM that consists of the two treatments: (1) the search treatment in the context of SDM under uncertainty, where the decision makers are not disclosed the payoff structure; (2) the choice treatment in the context of SDM under risk, where the decision makers are disclosed the payoff structure. The participants of the experiment participate in the search treatment first, then the choice treatment subsequently. The basic task is a binary choice among two alternatives—one is risky but higher EV and the other is safe alternative with lower EV—with the repetition of hundreds of trials. In the search treatment, the decision makers first make their choices without receiving any prior information on the possible outcomes and probabilities. In the choice treatment, the decision makers are informed of the information on the payoff structure. This paper studies behaviour when the difference in expected payoffs is quite small for a binary choice, and this choice task is repeated for hundreds of times. The results of the search treatment show that the tendency to select best reply to the past, and misestimation of the payoff distribution lead to robust deviations from maximisation. The results of the choice treatment show that the risk-averse decision makers engage in a substantial amount of mixing between a risky alternative and a safe alternative over hundreds of times. This paper proposes a model in which it is optimal for the decision makers to mix.

KEYWORDS: expected utility, experiment, sequential search, small decisions, prospect theory

Dr. Takemi Fujikawa
Research Fellow
Centre for Policy Research and International Studies
Universiti Sains Malaysia
Penang
Email : takemifujikawa@gmail.com

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1 Introduction

This paper presents experimental research on individual behavioural tendencies in "small feedback-based decisions", hereafter referred to as *small decision making* (SDM). Three critical features of SDM are presented in Fujikawa (2007b). First, it involves repeated tasks; the decision makers (DMs) face the same choice problem many times in similar situations. A second feature is that each single choice is of little consequence in terms of the expected value (EV); the options tend to have similar EV that may be fairly small. Barron and Erev (2003) state that the DMs invest little time and efforts in SDM. A third feature is that, in choosing among possible options, the DMs will have to rely on the immediate and unbiased feedback obtained in similar situations in the past. SDM is made without a careful evaluation of the possible payoff distributions. In SDM, the DMs are either disclosed (Fujikawa, 2005, 2008) or undisclosed (Barron & Erev, 2003; Erev & Barron, 2005; Fujikawa & Oda, 2007) a complete description of trivial choice problems. All of the authors asked the experimental subjects to make selections many times (i.e., a repeated-play paradigm). This paper presents a typical example of small decisions that we everyday face, and "non-trivial" impacts of small decisions on economic and social outcomes. Mainstream behaviourists (e.g., Allais, 1953; Kahneman & Tversky, 1979) focus on binary "big description-based decisions", hereafter referred to as *big decision making* (BDM) involving considerable differences in expected payoffs, and the decisions are not repeated.

On the contrary, the research on which this paper is based is solely on SDM that received little attention from researchers. Despite the significance of SDM, at the very outset, economists previously ignored SDM by assuming it to be irrelevant since—by definition—SDM has little immediate effects on the DMs, even though many common activities include SDM. However, the profession has been increasingly relying on SDM to explain many important phenomena. Since models of SDM have been extensively examined in the work of the creators of the field (Barron & Erev, 2003; Erev & Barron, 2005), their ideas have been further extended in the context of traditional economic theories, such as expected utility theory (EUT) (Fujikawa, 2006) and Bayes' rule (Fujikawa, 2007b). In addition to traditional economic theories, the idea of SDM is now applied to new areas, such as "Economics of Education" (Fujikawa & Gangopadhyay, 2006) and "Cultural Economics" (Fujikawa, 2005). It has been empirically documented that the DMs in SDM behave differently to their usual behaviour in BDM. This difference in behaviour is due mainly to changes in preferences of the DMs towards events with small probabilities. As a result, each DM's small decision can lead to collective outcomes that cannot be sustained by BDM. One implication is that the social interpretation of these small decisions by the DMs can lead to "non-trivial" impacts of small decisions on economic and social outcomes.

Here, attention is given to an important difference between risk, uncertainty and ambiguity: (1) A decision is made under *risk* when probabilities that certain states will occur in the future are precisely, completely known (e.g., in a fair roulette game); (2) A decision is made under *uncertainty* when the probabilities are not known (e.g., outcomes of sports events, elections or most real investments); (3) A decision is made under *ambiguity* when the probabilities are incompletely known (e.g., a bet on well-known Ellsberg's (1961) ambiguous urn). For an analysis performed throughout of this paper, this name convention concerning a degree of uncertainty is employed.

This paper presents the current experiment that consists of the two treatments: the *search treatment* for SDM under uncertainty, where the DMs are not disclosed payoff structure; the *choice treatment* for SDM under risk, where the DMs are disclosed exact payoff structure, prior to the experiment. A first part of this paper presents the search treatment that is an easily replicated design of the experiments implemented by Barron and Erev (2003), and provides new evidence and insight with respect to the mechanism of individual search propensity. A second part of this paper presents the choice treatment which is implemented to compare behavioural tendencies in SDM under uncertainty and behavioural tendencies in SDM under risk. Results of the choice treatment are analysed in the context of EUT as objective probabilities are available to the DMs. Among both treatments, the DMs face the repeated-play binary choice problems. The basic task of the DMs is a binary choice among two alternatives—one is risky but higher EV and the other is safe alternative with lower EV—with the repetition of hundreds of trials. An extensive analysis shall be made to answer the three central questions:

- Does the DM correctly search and estimate the payoff distribution of petty alternatives in terms of net payoffs with only hundreds of trials?
- Does EUT explain behavioural tendencies in SDM with the concave model?
- Does the exploration tendency observed in search tasks explain as rational exploration?

The new findings of this paper are as follows. In the search treatment, as Fujikawa and Oda (2007) theoretically show, the probability that the DMs *misestimate* the payoff structure of uncertain outcomes is fairly large in hundreds of rounds. This misestimation may lead the DMs to deviations from maximisation. The current results suggest that the DMs are likely to misestimate the payoff distributions in such a way that they consider the alternative with higher (lower) EV as the alternative with lower (higher) EV. From this view point, it is sceptical to ascertain Barron and Erev's (2003) claim that the DMs in their experiments were risk-averse with an observation of deviations from maximisation. Rather, the DMs just played best reply to the past, that is, they might merely choose the alternative more frequently that had produced high points *ex post*, so that the DMs misestimated each alternative. In the choice treatment, observed were the DMs who chose both a risky alternative with higher EV and a safe alternative with lower EV during given hundreds of trials. This seemingly puzzling behaviour can be explained by the expected utility (EU) models that are proposed in this paper.

The remainder of this paper contains the following. Section 2 provides a divergence between SDM and BDM. Section 3 reviews early work on SDM under uncertainty. Experimental design is included in section 4. Section 5 presents the search treatment. Section 6 presents the choice treatment. Finally, I conclude.

2 Divergence between SDM and BDM

As stated in an article by Barron and Erev (2003) and my previous article (Fujikawa, 2007b), it is imperative to uncover behavioural tendencies in SDM on the ground that nowadays many common activities involve SDM. For example, driving a car to work every morning requires repeated selections among speeds and various other options (e.g., whether or not to operate a stereo, whether or not to turn air condition on, and so forth, while driving). It may be reasonable for one driver to assume that EV of each selection and its difference are fairly small. One example of the driver's repeated selections is that she/he may suppose small difference in terms of EV in between an action to operate car stereo and an action not to operate it. Another example is concerned with routes. When the driver finds traffic jam with a chain of cars ahead of the road she/he is driving on, it seems SDM for her/him to decide whether to switch to another route (i.e., a shortcut) or to keep running on the same road. One may assume that the driver is allowed to invest very little time and efforts to make such a decision while driving. It follows that she/he, in an instant, cannot expect a total length of a chain of cars (i.e., she/he cannot anticipate whether it is just a slight traffic jam or heavy traffic jam).

Although little time and efforts are typically invested in SDM, it can be consequential. For example, accumulating a series of SDM in driving may lead to serious traffic accidents. The accidents are costly. In fact, Blincoe (1994) pointed out that the estimated cost of traffic accidents in US was more than 100 billion dollars a year. Furthermore, there has been a growing debate about car accidents caused by a series of SDM (e.g., answering a mobile phone in driving). For example, George E. Pataki—the former Governor of New York—banned in 2001 all New York State employees from using state-issued mobile phones while driving. Furthermore, a state employee driving a state vehicle was prohibited from using any mobile phones. There was the ban on the use of mobile phones while driving after Mr. Pataki's statement "Too many families have suffered the tragedy of seeing a loved one injured—sometimes fatally—in an accident caused by someone who was driving while using a cell phone" (The Office of the Governor, 2001).

There are two significant differences between SDM and BDM. The first difference is that SDM is generally studied in a "repeated-play feedback-based" paradigm with providing experimental subjects with *real* payoffs (i.e., cash payoffs) contingent on their performance in experimental tasks (Barron & Erev, 2003; Erev & Barron, 2005; Fujikawa, 2005, 2007a, 2007b, 2008). On the contrary, BDM is generally studied in a "one-shot description-based" paradigm with *hypothetical* payoffs, such as questionnaires (Allais, 1953; Kahneman & Tversky, 1979). Fujikawa (2007b) notes that what the DMs say they would do in hypothetical situations does not necessarily correspond to what they actually do. The second difference between SDM and BDM is that the DM in BDM takes a decision that is known to have a significant bearing on her/his welfare via the well-defined return and cost functions. However, the DM in SDM often relies on the subjective evaluation of possible payoff distributions—such as Bayesian updating and reinforcement learning—rather than her/his well-defined return and cost functions.

3 Literature review

Greg Barron and Ido Erev—the creators of the field of SDM—conducted an interesting series of experiments on SDM under *uncertainty*. Barron and Erev (2003) reported some properties of SDM under uncertainty, such as deviations from EV maximisation, which are the reverse phenomena observed in BDM. Their results revealed that the DMs' experience led the *reversed* certainty effect, contrary to the certainty effect that is the pervasive tendency in BDM presented by Kahneman and Tversky (1979). Erev and Barron (2005) showed that the DMs were likely to be sensitive to payoff variability and payoff ranks in making decisions in SDM under uncertainty. Erev and Barron analysed the payoff variability effect and the payoff rank effect, applying to reinforcement learning among cognitive strategies. Erev (2007) introduced laboratory research that examined the basic properties of SDM under uncertainty that were made based on the DMs' personal experience. It suggested that the DMs' decisions from their experience were very different from decisions that were made based on precise descriptions of the possible payoff distributions.

My earlier paper (Fujikawa, 2007b) contained an experimental investigation of SDM under *ambiguity*, and discussed decisions from experience to Bayesian updating. Fujikawa (2007b) presented an experimental treatment in which the DMs' priors were manipulated to be optimistic and pessimistic. That is, there were two states of nature in the treatment: a favourable state and an unfavourable state, but only one of them obtained on any given trial. The significant effect of manipulation emerged when two conditions were met: (1) one of alternatives of binary choice problems included uncertain prospect with higher EV; (2) both alternatives included risky prospect.

A present analysis complements a substantial body of previous experimental research conducted by Greg Barron and Ido Erev. Hence, it is worthwhile to describe their work briefly here. In their articles (Barron & Erev, 2003; Erev & Barron, 2005), the authors implemented the following Problem 1, Problem 2, and Problem 3:

Problem 1. Choose between:

H: 4 points with probability 0.8 ; 0 otherwise

L: 3 points with certainty

Problem 2. Choose between:

H: 4 points with probability 0.2 ; 0 otherwise

L: 3 points with probability 0.25 ; 0 otherwise

Problem 3. Choose between:

H: 32 points with probability 0.1 ; 0 otherwise

L: 3 points with certainty

The procedural paradigm involved a choice between two unmarked buttons on a computer screen to which each DM was assigned. In each trial t (t=1, 2, … 400), the DM was asked to click on one of the two buttons. Each click corresponded to the payoff associated with the selected button. The DM received no prior information concerning the relevant payoff distributions, but could see the drawn value (the obtained payoff) after each trial on her/his computer screen. That is, the information available to the DM was limited to feedback concerning the outcomes of her/his previous decisions. Note that, in Kahneman and Tversky's (1979) experiment, the DMs were correctly disclosed the payoff structure, and they performed only one round in Problem 1 and Problem 2 with hypothetical payoffs. On the other hand, Greg Barron and Ido Erev (Barron & Erev, 2003; Erev & Barron, 2005) carried out experiments, where the DMs were undisclosed the payoff distribution, asked to choose 400 times, and paid real money according to their performance at a conversion rate of one point to 0.01 Israeli Shekel (about 0.25 US cent at the time of the experiment).

It was claimed in Barron and Erev (2003) and Erev and Barron (2005) that the *reversed* certainty effect was observed in their experiments. While the average proportion of H choices over the subjects was 0.63 for Problem 1, it decreased significantly to 0.51 for Problem 2. However, it is not safe to compare directly their results with Kahneman and Tversky's (1979) results due to the following two caveats which are stated against statements in Barron and Erev (2003) and Erev and Barron (2005).

First, it has not been examined in Barron and Erev (2003) and Erev and Barron (2005) whether or not the DMs could have correctly estimated each alternative only with hundreds of trials.[2]

[2] One might have been willing to conduct a questionnaire at the conclusion of the experiments to ask the DMs whether or not they correctly estimated payoff structure.

As they did not have prior information as to the payoff structure, the DMs would have to refer to feedback of their past outcome in every round to estimate the payoff distribution of the choice problems. In the process of trying alternatives repeatedly, the DMs would gradually form their subjective payoff distribution of the problems, which was or was not the same as the objective payoff distribution. Having finished searching for the payoff distribution of both alternatives (H and L) with only 400 trials, some DMs subjectively judged H (L) as the alternative with higher (lower) EV; that is, they had estimated the alternatives correctly. Others, however, subjectively judged H (L) as the alternative with lower (higher) EV; that is, they had misestimated the alternatives. Therefore, it is quite ambiguous that the DMs chose H (L) supposing it yielded higher (lower) EV.

Second, an optimal behaviour for a choice problem with repeated-play tasks is not necessarily to repeat an optimal behaviour for a choice problem with one-shot tasks. Suppose that, for example, the DM is willing to choose H in Problem 1 when she/he is asked to perform Problem 1 only once. This does not necessarily imply that this DM is willing to choose H 400 times in Problem 1 when she/he is asked to perform Problem 1 400 times.

As discussed above, we must note that the experiments conducted by Greg Barron and Ido Erev are quite distinct from Kahneman and Tversky's (1979) experiment. Barron and Erev (2003) present the reversed certainty effect by comparing the results of their experiment (Experiment 2 in Barron and Erev (2003))—with the results of Kahneman and Tversky's experiment (Problem 3 and Problem 4 in Kahneman and Tversky,(1979)). Yet, Barron and Erev's claim must be carefully interpreted because their results should not be directly comparable with Kahneman and Tversky's results.

4 Experimental design

This section describes an apparatus and procedure of the current experiment that includes two treatments: the search treatment for SDM under uncertainty and the choice treatment for SDM under risk. In the search treatment, the DMs were disclosed *neither* prior information on possible outcomes and probabilities, *nor* the exact length of the experiment. Yet, the DMs were disclosed that they were to perform several choice problems. On the other hand, the choice treatment was conducted under the condition that the DMs were disclosed precise information on possible outcomes and probabilities, and the exact number of rounds to be performed. Please note that, when recruited, the DMs were not informed that they were required to perform two treatments. Instead, they were informed that they were required to play several experimental treatments, and an estimated duration of those was two hours. Both treatments were computerised and conducted at the Kyoto Experimental Economics Laboratory (KEEL) in Japan with 42 paid subjects—undergraduates from various faculties at Kyoto Sangyo University. The DMs participated in the search treatment first, and participated in the choice treatment subsequently. On their arrival at the KEEL, each DM was assigned a workstation that displayed an experimental screen, and distributed only a written instruction of the search treatment, as they were asked to start with the search treatment. (The instruction and experimental screen are available in Appendix.) The instruction was read aloud and the DMs were given an opportunity to ask questions individually.

The DMs were confronted with Problem 1, Problem 2, and Problem 3 in each treatment. The DMs' task was to make a selection between H and L at each round t (t=1, 2, … , 400) in each problem in the two treatments. They were instructed to operate a "computerised money machine" in the two treatments. The DMs in the search treatment were instructed to choose one of two *unmarked* buttons shown in Figure 1 which corresponded to H and L for 400 times in each of the three problems. All of the same types of the procedure explained above were employed for the choice treatment, except that the DMs in the choice treatment were presented with two *marked* buttons shown in Figure 2 on which corresponding payoff and its probabilities were appeared. Amongst both treatments, the money machine provided the DMs with binary types of feedback immediately following each choice: (1) the payoff for the choice that appeared on the screen for the duration of one second, and (2) an update of an accumulating payoff counter, which was constantly displayed. A protocol of the experiment was as follows:

- At first, the DMs played Problem 1, Problem 2 and Problem 3 in the search treatment; that is, they were played 1200 trials in the search treatment (400 trials for each problem). As noted above, the DMs in the search treatment were not informed that they were to play exactly three choice problems, in each of which the DMs were presented with a 400-fold repetition of a binary choice. Hence, the DMs were not aware that they had had 1200 trials to play in the search treatment. Instead, they were aware that they faced several choice problems in the search treatment. An experimental procedure was that the DMs started with Problem 1 and made 400 selections in Problem 1. Then, the DMs were prompted to move to Problem 2 by the automatically-generated message on the screen on their completion of Problem 1. (The message is presented in the instruction that is available in Appendix.) Hence, they were aware when a change from Problem 1 to Problem 2 was generated; that is, on their completion of Problem 1, they were advised that Problem 1 had been completed and they moved on Problem 2. The same procedure applied to when a change from Problem 2 to Problem 3 was generated.
- Having confirmed that all DMs in the laboratory had finished the search treatment, the experimenters distributed a written instruction of the choice treatment that explained a protocol of the choice treatment, and the payoff structure of three choice problems to be played in the choice treatment. The instruction explained that, in the choice treatment, the DMs were asked to play the three choice problems, each of which consisted of 400 trials. (The written instruction is available in Appendix.) The instructions were read aloud, and the DMs were given an opportunity to ask questions individually. Then, the DMs were asked to start the choice treatment following the same experimental procedure as the search treatment.
- At the conclusion of the experiment, the DMs were paid individually and privately at a conversion rate of one point to 0.3 Yen (about 0.25 US cent at the time of the experiments), and received no initial (showing up) fee.

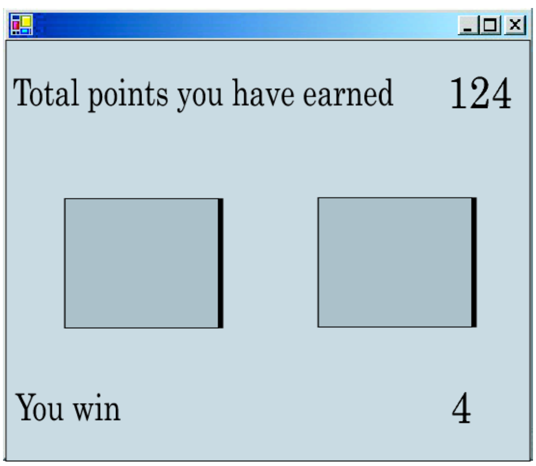

Figure 1 Computerised money machine for the search treatment

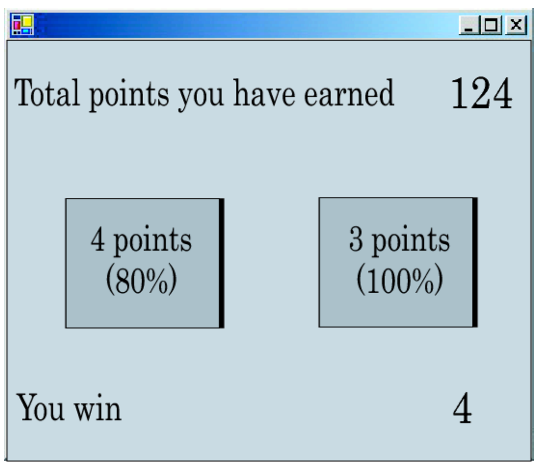

Figure 2 Computerised money machine for the choice treatment

5 The search treatment

5.1 Results

The overall maximisation rate (*choiceH*) is 0.48, 0.55 and 0.22 for Problem 1, Problem 2 and Problem 3 respectively. It follows that H, for example, was chosen on average 192 out of 400 times in Problem 1. We here define the "posterior average points of H choices" (*posteriorH*) as the points the DM has earned from H after it was chosen *m* times. For example, if the DM has gained 12 points from H after she/he chose it five times, then her/his *posteriorH* is 2.4(=12/5). The results of the search treatment reveal that the overall empirical *posteriorH* becomes less than expected *posteriorH*. The empirical *posteriorH* becomes 2.88, 0.78 and 1.74 for Problem 1, Problem 2 and Problem 3 respectively. Please note that expected *posteriorH* is 3.2, 0.8 and 3.2 for Problem 1, Problem 2 and Problem 3 respectively. Figure 3-5 illustrate *choiceH* and *posteriorH* for each problem.

Problem	H	L
1	(4, 0.8)	(3, 1)
2	(4, 0.2)	(3, 0.25)
3	(32, 0.1)	(3, 1)

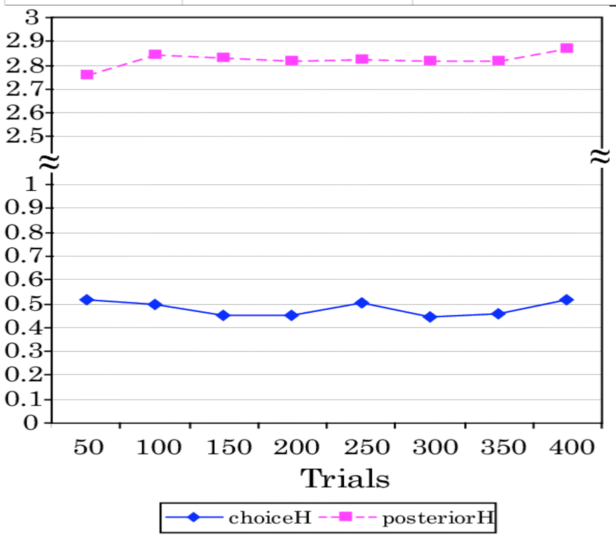

Figure 3 *choiceH* and *posteriorH* in Problem 1

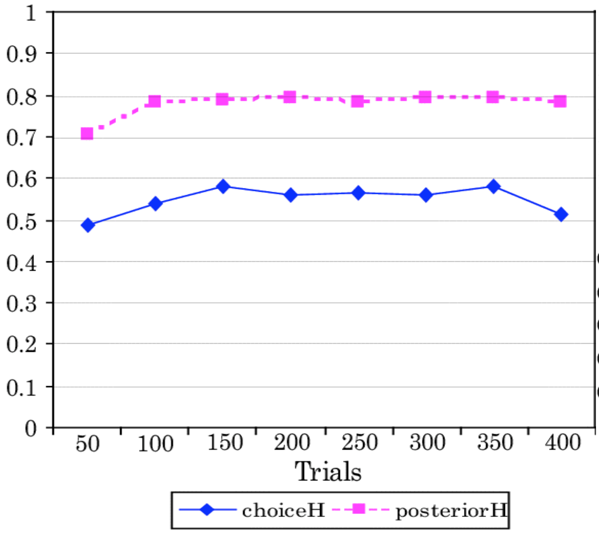

Figure 4 ***choiceH*** **and** ***posteriorH*** **in Problem 2**

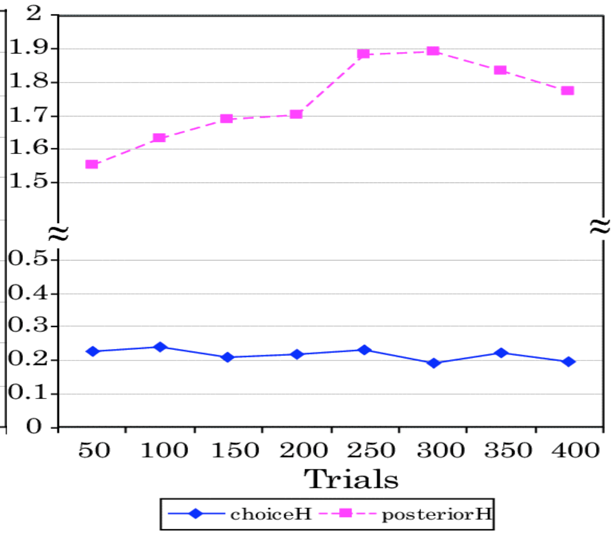

Figure 5 ***choiceH*** **and** ***posteriorH*** **in Problem 3**

5.2 Analysis

The current results reveal that *choiceH* for Problem 1 and Problem 3 has become less than 0.5: *choiceH* for Problem 1 and Problem 3 was 0.48 and 0.22 respectively. This seemingly puzzled tendency that the DMs overall chose L more than half could result from the DMs' insufficient search with only a few hundreds of trials and therefore their misestimation of the alternatives.[3] This section reviews the *search-assessment model* presented in Fujikawa and Oda (2007) to show how large is the probability that the DMs would misestimate the payoff structure of the two alternatives.

5.2.1 The search-assessment model

The search treatment includes the situation in which the information available to the DMs is limited to feedback concerning the outcomes of their previous decisions. In this situation, the DMs will have to search both of the two alternatives, H and L, to discover the objective payoff structure of the alternatives.

Suppose that the DM is asked to choose either H or L in the following Problem X at each round t (t=1, 2, … , T):

Problem X. Choose between:

H: x (p) ; 0 (1-p)

L: 1 (1)

where $p \in (0,1)$ and $px>1$. In Problem X, one selection of H enables the DM to earn x points with probability p, and zero point with (1- p); one selection of L enables her/him to earn one point for sure. To analyse Problem 1 and Problem 3, where only one alternative includes uncertain prospect, we shall

[3] In the search treatment, the DMs had to try out both H and L to search the payoff structure of both alternatives, so that the results might reflect the DMs' unwilling choice of L taken for merely search. Yet, even if they had to choose H and L alternatively to discover payoff structure in the first half of the search treatment, the EV-maximising DMs would have chosen H more frequently in the latter half if they regarded H as the alternative with higher EV.

examine Problem X, which applies to Problem 1 by setting p=0.8 and x=4/3, and applies to Problem 3 by setting p=0.1 and x=32/3.

Let W be the probability that the DM judges that H has higher EV than L after she/he has chosen both alternatives. In other words, W is the probability that the DM's *posteriorH* becomes greater than or equal to one, which is EV of L. Then, we can express W as follows:

$$W = \sum_{\text{all } m:\frac{kx}{m}\geq 1} {}_mC_k p^k (1-p)^{m-k} = \sum_{k=\left\lfloor \frac{m}{x} \right\rfloor +1}^{m} {}_mC_k p^k (1-p)^{m-k} \tag{1}$$

where k is the number of the highest payoff of H realised, and ${}_mC_k$ is a binomial coefficient, that is, the number of ways of picking k unordered outcomes from m possibilities.

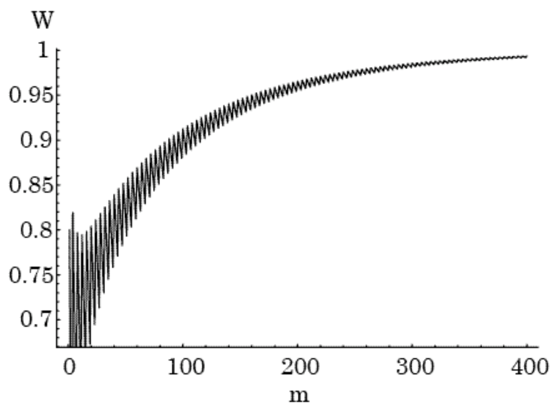

Figure 6 *W* for Problem 1

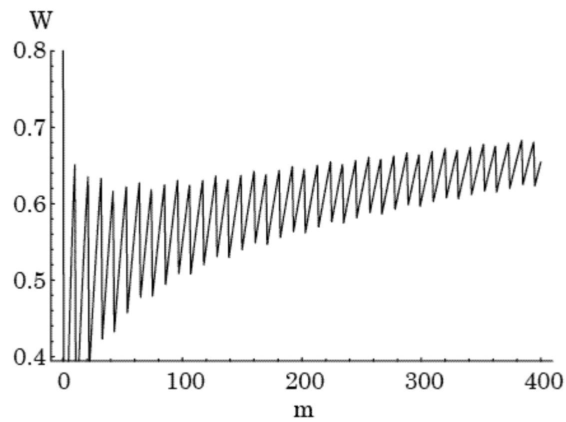

Figure 7 *W* for Problem 3

Figure 6-7 depict W for Problem 1 and Problem 3 respectively. If the DM has chosen H 200 times in Problem 1, then her/his *posteriorH* exceeds 3 with probability of 0.97 as shown in Figure 6. On the other hand, Figure 7 shows that even though the DM has chosen H 200 times in Problem 3, her/his *posteriorH* exceeds 3 with probability of 0.63. Interestingly, W does not exceed 0.98 until the DM has chosen H 10,000 times in Problem 3.

Next, let us examine Problem Y to analyse Problem 2, where both of the two alternatives—H and L—include uncertain prospect:

Problem Y. Choose between:

H:	x	(θp)	;	0	($1-\theta p$)
L:	1	(θ)	;	0	($1-\theta$)

where $p,\theta \in (0,1)$ and $\theta px > \theta$. The DM in Problem Y is asked to choose either H or L at each round t (t=1, 2, … , T). Problem Y applies to Problem 2 by setting p=0.8, θ=0.25 and x=4/3.

Let Z be the probability that *posteriorH* becomes equal to or greater than *posteriorL* after the DM has chosen H and L m and n times, respectively. Then, Z is formulated as

$$Z = \sum_{k=0}^{m} \left[{}_mC_k(\theta p)^k (1-\theta p)^{m-k} \times \sum_{j=0}^{\left\lceil \frac{nkx}{m} \right\rceil} {}_nC_j \theta^j (1-\theta)^{n-j} \right]. \tag{2}$$

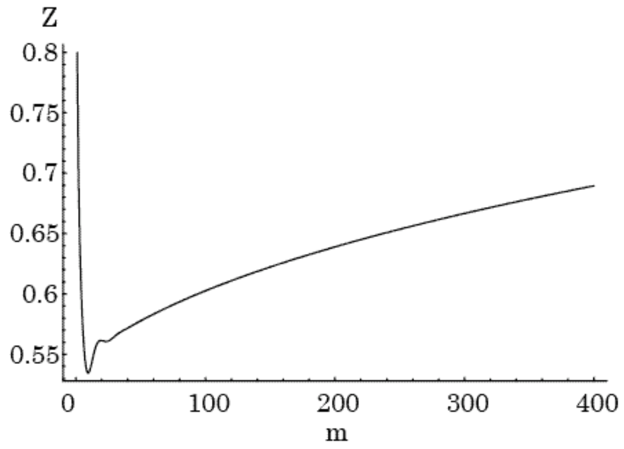

Figure 8 *Z* for Problem 2

Calibration of Z maintains that in choosing H and L each 200 times in Problem 2, the DM will judge that H has higher EV than L with probability of 0.64 as shown in Figure 8.

5.2.2 Experimental

This section describes that the DM's selection toward each alternative is *not* dependent upon only the characteristics of her/his utility function *but* her/his *posteriorH* and *posteriorL*. In fact, there was a considerable difference of *posteriorH* among the DMs in the search treatment. For further discussion, the DM's tendency toward each alternative is considered from the following two hypotheses. First, individual risk attitude is most important subjective factor for the DMs in SDM under uncertainty. It follows that the DMs make their decisions depending upon individual risk attitude: "risk-seeking" or "risk-averse". For example, let us focus on one DM observed in the experiment. She chose L at most of rounds in Problem 1, and her *posteriorH* was greater than three, which is EV of L. It is safe to consider that this DM was risk-averse because she preferred L to H despite that her *posteriorH* was greater than EV of L. On the contrary, another DM, who frequently chose H in Problem 3, was observed in the experiment. Her *posteriorH* became less than three. It makes sense that this DM was risk-seeking because she preferred H to L despite that her *posteriorH* was less than EV of L.

The second hypothesis is that the DMs make decisions, playing best reply to the past outcomes. This hypothesis stipulates that the DMs make decisions depending highly upon their *posteriorH* or *posteriorL*. For example, the DMs' *posteriorL* over 400 trials in Problem 1 was of course 3, while their overall empirical *posteriorH* became 2.88 in Problem 1. It means that some DMs' *posteriorL* exceeded *posteriorH* for Problem 1. This seemingly paradoxical consequence has been examined by "the search-assessment model" developed by Fujikawa and Oda (2007). It seems thereby reasonable for the DMs to choose L frequently (192 out of 400 times) in Problem 1 so as to maximise their payoff, taking into their subjective consideration that L would have higher EV than H. For example, one DM was observed in the experiment: her *posteriorH* became 2.29, and she chose H only 14 times (3.5%) in Problem 1. We, however, do not guarantee that this DM was risk-averse and had a concave utility-of-wealth function. It is safe to presume that her *posteriorL* had become greater than *posteriorH*, and she chose L many times. Therefore, it is inevitable to note that *posteriorH* and *posteriorL* will, more or less, affect the DMs' decisions.

As discussed above, several DMs in Problem 1 chose frequently L—the safe alternative. They were, however, not necessarily risk-averse. It is quite likely that they chose L many times as a result of their misestimation of the payoff structure. The search-assessment model presented in Fujikawa and Oda (2007) asserts that very high is the probability that the DMs could misestimate the payoff distributions of the alternatives in just hundreds of rounds. In fact, several DMs were observed, who chose L due to the fact that their *posteriorL* became greater than *posteriorH.* Here is the presumption that they would have expected to maximise their payoff by choosing L in subsequent rounds, as L had yielded higher points than H *ex post*. This presumption conforms to the discussion stimulated in Bruni and Sugden (2007) that if different actions in an experimental environment consistently lead to different monetary payoffs, laboratory subjects who are motivated to maximise their own payoffs can learn to choose actions which are in fact payoff-maximising. Although Barron and Erev (2003) did not mention the existence of their DMs' misestimation of the payoff structure, we should draw careful attention to what extent the DMs' decision is affected by their misestimating of the payoff structure in SDM.

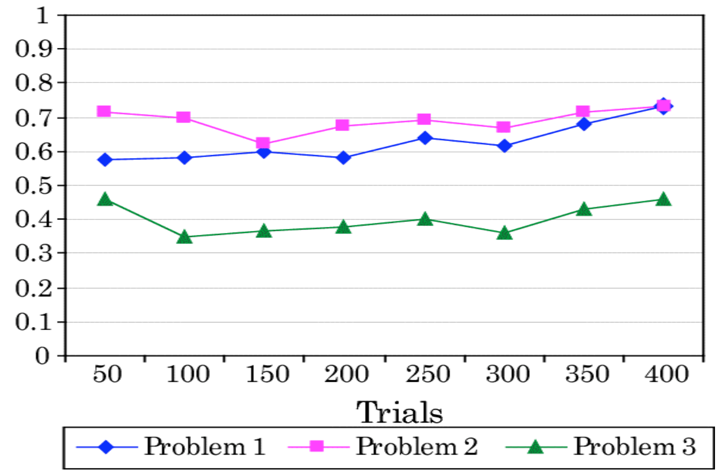

Figure 9 *choiceH* in the choice treatment

6 The choice treatment

6.1 Results

Results of the choice treatment reveal that *choiceH* is 0.63, 0.69 and 0.40 for Problem 1, Problem 2 and Problem 3 respectively, and illustrated in Figure 9. It is found from the results that the DMs often chose both H and L in all problems. The results also reveal the existence of heterogeneity among the DMs, regarding behavioural tendencies in the three problems. For example, one DM was observed, who chose H 400 times (100%) in Problem 1 and chose H 268 times (67%) in Problem 2.

6.2 Analysis

In the choice treatment, the DM was asked to choose either H or L at the *t*-th (*t*=1, 2, … , 400) round with the knowledge of the past outcome except for the first round. An outcome from past choices did not serve her/his knowledge, as the DM was provided with full information on the payoff structure in the choice treatment. Every DM had the full knowledge of the payoff structure of the alternatives from the beginning of the experiment, as she/he was disclosed the information prior to the experiment. Although there may exist "asset effect" in repeated tasks, the present discussion will unravel the behavioural tendencies in the experiment, assuming that the DM performs repeatedly 400 rounds without being informed of the outcome of her/his past choice, and she/he is asked how many times out of 400 rounds she/he is willing to choose H or L once for all.[4]

6.2.1 Expected utility models

Results of the choice treatment can be analysed within the framework of EUT since the DM was disclosed an objective payoff structure prior to the experiment. Making objective probabilities of alternatives available to the DM allows direct evaluation of EUT. This section presents the EU model to show that the DM with a risk-averse utility function (i.e., a concave utility-of-wealth function) should choose the risky alternative in most of the rounds, but *not all the time* within given trials, so as to maximise her/his EU. The model shows that it is theoretically-optimal to mix between the risky alternative and the safe alternative for the risk-averse DM. It follows that the DM can maximise her/his EU by choosing the risky alternative for particular times within given trials (400 times in each problem).

Here, we assume the concave model in which the DM's utility function is strictly increasing and concave. Employed is the utility function, $u(x)$, to examine that the DM should choose the risky alternative for specified times to maximise her/his EU in Problem 1 and Problem 3. Note that $u(x)$ is a utility function defined on a sure amount of money, \$$x$; that is, $u(x)$ is the DM's utility from \$$x$. Letting a, R, x, α, β, γ, δ, $\theta>0$ and $\alpha>\beta>\gamma>\delta$, $u(x)$ is the risk-averse utility function for plausible parameters:

$$u(x) = \frac{\ln(x+R^{\theta})e^{ax} - (x+a)^{\alpha}}{(x+R^{\eta})^{\beta} + x^{\gamma} + x^{\delta}}. \tag{3}$$

For concavity,

$$\frac{du'(x)}{dx} > 0 \text{ and } \frac{du''(x)}{dx} < 0. \tag{4}$$

Next, we let $V_1(m)$ and $V_3(m)$ be the EU the DM acquires when choosing H m ($0\leq m\leq 400$) times in Problem 1 and Problem 3 respectively:

$$V_1(m) = \sum_{k=0}^{m}\left[{}_{m}C_{k}(0.8)^{k}(0.2)^{m-k}u(1200-3m+4k)\right] \text{ and} \tag{5}$$

$$V_3(m) = \sum_{k=0}^{m}\left[{}_{m}C_{k}(0.1)^{k}(0.9)^{m-k}u(1200-3m+32k)\right] \tag{6}$$

where k is the number of the highest payoff of H realised in Problem 1 and Problem 3. As we have seen, the DMs made their decisions by choosing *both* H and L in Problem 1 and Problem 3. Calibration of $V_1(m)$ and $V_3(m)$ figures a theoretically-optimal number of H choices for the risk-averse DMs in Problem 1 and Problem 3.

[4] Barron and Erev (2003) state that "one-at-a-time" repeated choices are distinct from "all-at-a-time" choices in SDM under *uncertainty*. Yet, this assumption is not refuted but oriented in analysing behaviour in SDM under *risk*, where the DMs receive clear, prior information as to the payoff structure.

First, we investigate an optimal behaviour in Problem 1 by conducting calibration of $V_1(m)$ with $u(x)$ for a=0.001, R=10, α=0.05, β=0.1, γ=0.5, δ=0.5, θ=-0.05 and η=6. Figure 10 shows that $V_1(m)$ with these parameters has its maximum at m=252 for $0 \leq m \leq 400$. It follows that a theoretically-optimal number of H choices for the DMs in Problem 1 is 252. That is, the DMs maximise their EU by choosing H 252 out of 400 times. This overwhelmingly reveals the DMs behaviour in the choice treatment. Recall that *choiceH* for Problem 1 is 0.63, indicating that H was chosen 252 times on average.

Next, we pursue calibration of $V_3(m)$ to explore the DMs' optimal behaviour in Problem 3. The calibration is conducted with $u(x)$ for a=0.001, R=10, α=0.05, β=0.1, γ=0.5, δ=0.5, θ=-0.05 and η=4. It is found that, as shown in Figure 11, $V_3(m)$ has its maximum at m=169 for $0 \leq m \leq 400$. We elucidate that a theoretically-optimal number of H choices for the DMs in Problem 3 is 169. It maintains that the DMs maximise their EU by choosing H 169 out of 400 times in Problem 3. This well reflects the DMs' behaviour in the choice treatment. Recall that *choiceH* for Problem 3 is 0.4, following that H was overall chosen 160 times in Problem 3.

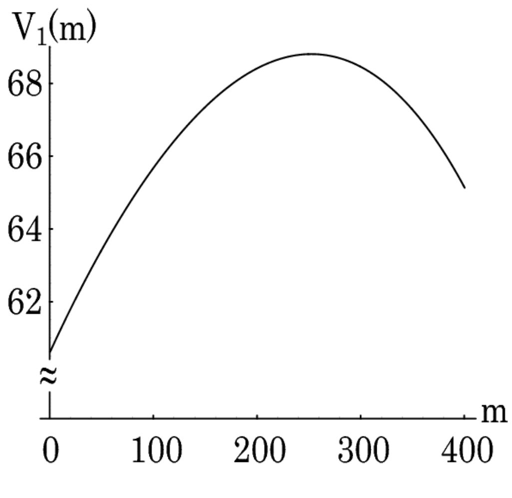

Figure 10 EU for Problem 1

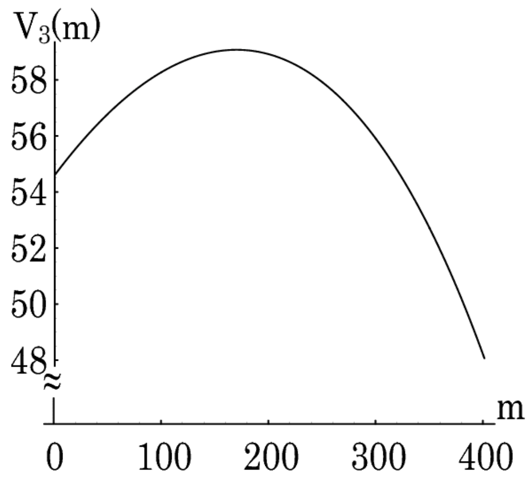

Figure 11 EU for Problem 3

7 Concluding remarks

This paper has revisited the roles of mechanisms of individual decision making in SDM under uncertainty and SDM under risk. The first part of the current paper presented an author's experimental analysis of SDM under uncertainty to complement an earlier work by Greg Barron and Ido Erev. Their work (Barron & Erev, 2003; Erev & Barron, 2005) showed developments in analysing individual behaviour in SDM under uncertainty. To develop their analysis, the search treatment was implemented. Results showed the demonstration that the tendency to select best reply to the past and misestimation of the payoff distribution could lead to robust deviations from maximisation. We found that the probability that the DM misestimates the probability of uncertain outcomes is fairly large after a few hundreds of trials (400 trials in each problem in the search treatment). This misestimation may lead the DM to deviations from maximisation. In fact, it is suggested that many DMs in the search treatment were likely to misestimate the payoff distributions in such a way that they considered the alternative with higher (lower) EV as the one with lower (higher) EV. The present paper asserts that the DM's search of the payoff structure of uncertain outcomes after only a few hundreds of trials is insufficient. None of the previous studies on SDM under uncertainty mentioned the existence of the

DM's misestimation of the payoff structure in experimental tasks. However, the problem of the DM's misestimation of the payoff structure due to her/his insufficient search should be carefully interpreted.

The second part of the current paper presented an author's experimental analysis of SDM under risk, which could provide new evidence on the way that the DMs' behaviour is explained in the context of EUT. For further investigation of the DMs' behaviour in SDM under risk, the choice treatment was implemented. Results showed that the DMs tended to choose both risky and safe alternatives within given trials. To account for this seemingly puzzling behaviour, the EU models were proposed to provide a theoretical analysis of prediction of EUT for a repeated choice paradigm. Calibration of the models has maintained that it is theoretically-optimal to choose the risky alternative often but not all the time for the risk-averse EU-maximising DMs. It follows that the DMs can maximise their EU by engaging in a substantial amount of mixing between the risky alternative and the safe alternative over the trials.

In presenting the current experimental results, this paper has employed the methodology that is to use the data/results in an aggregated form. This methodology is essentially identical to the one used in previous studies on individual feedback-based decisions (Barron & Erev, 2003; Erev & Barron, 2005; Fujikawa, 2005, 2007b). In analysing a large set of experimental results on feedback-based decisions (i.e., the results of the current experiment that includes repeated-play tasks, and each treatment consists of a 1200-fold repetition of choices), it is straightforward to present aggregated results to summarise a large set of data. In fact, the previous studies introduced above have presented a fruitful discussion that is based on experimental data and results in an aggregated form. As the present paper presents a series of experimental tasks, each of which includes hundreds of rounds of decision tasks, I have followed the methodology employed by the previous studies in unravelling behavioural tendencies in feedback-based decision making.

The current experimental setting exhibited the fact that the DMs first completed the search treatment, and then moved to the choice treatment. One might claim that this fact resulted in a lack of control over some behavioural variations: if the DMs' behaviour got less variable over time, due probably to their chronic nuisances (i.e., the DMs' fatigue and boredom), then this would affect the experiment in an asymmetric way. However, the current experimental setting seems congruent with standards of economics experiments in terms of duration of the experiments, and, henceforth, this claim shall be refuted. In the current experiment, the DMs were asked to remain at the laboratory for 90 minutes to accomplish a whole experimental procedure. This setting was created in line with Friedman and Cassar (2004) and Friedman and Sunder (1994). Friedman and Cassar state that experimenters should try to keep sessions no more than two hours, otherwise experimental data may reflect the DMs' tiredness. Accordingly, Friedman and Sunder recommend experimenters to implement experiments in which the DMs spend at most two hours to minimise their fatigue and boredom.

This paper has addressed an important issue on SDM. A series of disucussions presented in this paper have accounted for the contrast between SDM under uncertainty and SDM under risk. The current analysis may imply boundaries of the observations reported by Greg Barron and Ido Erev (Barron & Erev, 2003; Erev & Barron, 2005).

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Appendix A

Instruction

1 Introduction

Thank you very much for joining our economics experiment. In this experiment you are asked to play easy games. Your goal is to complete the experiment with as many points as possible. The more points you earn, the more cash you can receive. The procedure of this experiment is explained along this instruction.

2 Distributions

Please confirm whether you received the following four items:

- Instruction (This leaflet)
- Questionnaire form
- Receipt form
- Subject NO card

3 Receipt

Please write in the form your name, ID number, address, and the date today in advance. Note that keep the amount blank.

4 Notice

- You may NOT create a disturbance.
- You may NOT leave the laboratory during the experiment.
- You may keep switching your portable phone off during the experiment.
- You must leave all items distributed by personnel in the laboratory.
- You may NOT touch a keyboard.
- Do NOT click on right.
- You may NOT attempt to tamper with a computer.

Failure to comply with administrator's directions can result in points you earned being cancelled and no money will be paid.

5 If you need an administrator

If at any time during the experiment you believe you have a problem with your computer or need an administrator for any reason, raise your hand.

6 Payment

At the conclusion of the experiment, points will be converted to monetary payoff according to the exchange rate: 100points =30yen. The amount below 10 yen is rounded up.

7 Procedure

7.1 Registration

Check that Figure 1 is displayed on your screen. (If it is not, raise your hand.) Click on "►" or "◄" button on your screen in order to equalise the number appeared on screen with your subject number then press "Correct". Assuming that your subject number is I-19, press "Correct" in Figure 2.

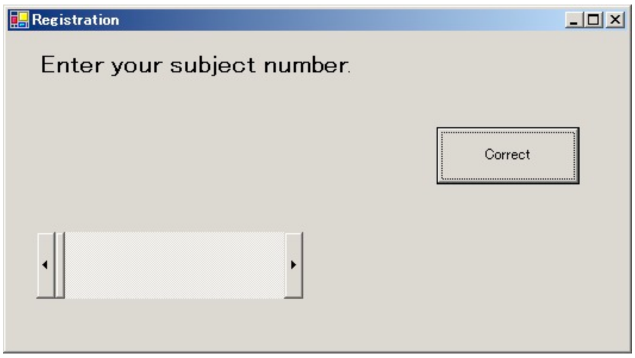

Figure 1

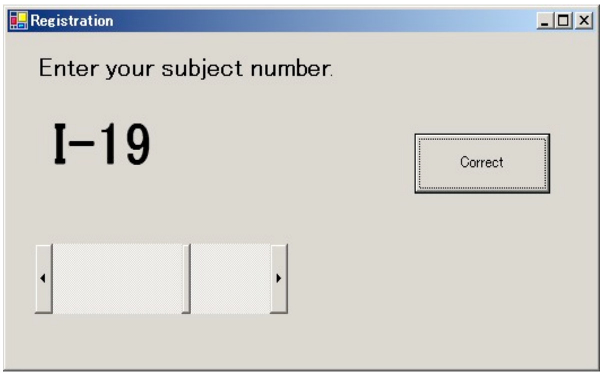

Figure 2

7.2 How to Operate

The experiment consists of two treatments: the former treatment, and the latter treatment. You perform the former and the latter in order. Each treatment consists of several sessions. Each session consists of several rounds. You are asked to choose either the right or the left button in each round as seen in Figure 3. The points corresponding to selected button appear on the right side of "You win" (see Figure 4 as an example) and you can get it at that round. Your income is calculated by the computer.

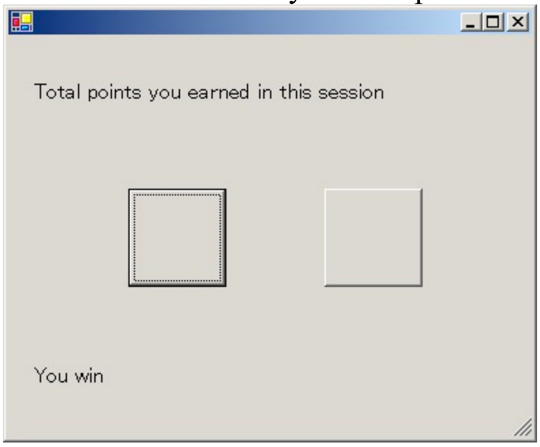

Figure 3

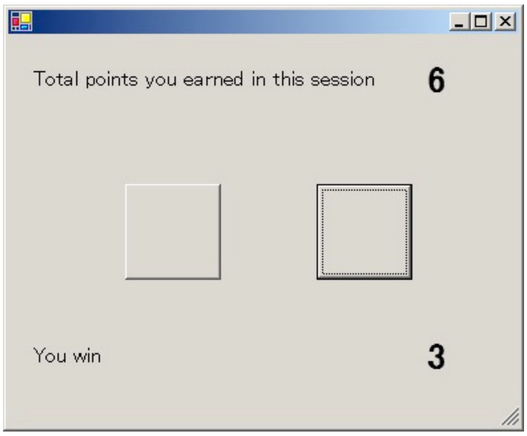

Figure 4

You are asked to play along this procedure for specific times in each treatment. Points are contingent upon the button chosen. The different session has the different structure of the experiment. Your score is not affected by other's behaviour. An update of an accumulating score is constantly displayed on the right side of "Total points you earned in this session". After completing each session, Figure 5 appears. Then Figure 6 appears after pressing "OK" in the Figure 5.

Figure 5

Figure 6

Press "OK" and you can face Figure 3 again. Begin to play new session as in the first session you did.

7.3 Conclusion of the former treatment

Figure 7

Figure 7 appears when the former treatment is over. An administrator will explain the procedure as to the latter treatment. Do NOT proceed to the next session until you are asked by an administrator to do so.

Please wait.

Appendix B

Instruction for the latter treatment

The latter treatment consists of four sessions. Each session consists of 400 rounds, where you are repeatedly faced with the same problem. In contrast to the former treatment, the payoff structure of each problem appears on each button. For example, if you face Figure 6 and choose the left button, then you get four points with probability 80% and 0 with probability 20%: if you choose the right button, you get three points for sure. Your decision in each round is mutually exclusive and equally likely. Note if the sample size consists of n mutually exclusive and equally likely outcomes, then the probability of any single outcome is 1/n.

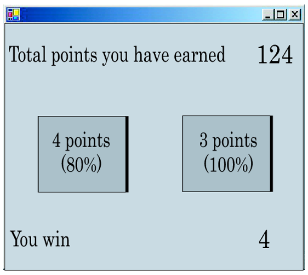

Figure 6

Figure 7

7.4 Conclusion of the experiment

When all sessions you are asked to play are finished, Figure 7 appears. Please click “OK”, then you could complete all the procedures of today’s experiment. Please wait until the administrator finishes preparing the rewards for you.